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Structural Damage Detection Based on Proper Orthogonal Decomposition: Experimental Verification

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The present paper describes a new structural damage detection method based on the monitoring of vibrational properties of the structure. Sensors record the accelerations of several points of the structure. The recorded data are then used to compute the proper orthogonal modes. The comparison between the proper orthogonal modes of the undamaged structure and those of the damaged structure provides information on damage location. Experimental results are presented.

I. Introduction

Structural components in different fields of engineering have to be in continuous use, despite aging and the unavoidable risk of damage growth and consequent failure. Damage is usually defined in terms of a comparison between two different states, one of which is assumed to represent the initial undamaged state, and the other is assumed to represent the current state of the system. Monitoring the structural health of engineering systems in a nondestructive way is becoming increasingly important, both for safety and economy reasons. A complete review of the different nondestructive testing (NDT) methods is given in [1].

Many NDT methods are usually used *locally* for the detection of defects concentrated in small portions of the structure. Therefore, the examination of the whole structure may require several applications of the nondestructive techniques. It is desirable to have *global* NDT methods to reveal, quickly and cheaply, the presence of damage in a zone of the structure to which a *local* technique would then be effectively applied for a more precise assessment. The variations of vibration properties were selected as damage indicators in the past and constitute one of the few damage indicators capable of monitoring the whole structure simultaneously [2,3]. Although vibration-based damage detection may appear intuitive, its actual application poses largely unresolved challenges. The reasons that the practical application of vibration-based methods is limited are manifold.

First, if modal methods are used to identify damage in a structure, it is usually necessary to develop a reliable mathematical model of the structure that correctly represents the damage [4,5], a task that is always difficult, and occasionally impossible, to perform. Furthermore, damage (in its initial phase, when it has to be detected) is typically a local phenomenon and may not significantly influence the low-frequency global response of a structure, which is usually measured in vibration tests.

Vibration properties, determined from the response of a structure, are mainly the natural frequencies and the corresponding mode shapes and damping factors. A survey on the use of natural frequency changes for damage detection is presented in [6], in which it is concluded that the shift in natural frequencies has some important practical limitations. The natural modes are the vibration properties that have to be monitored to not only detect, but to also locate, the presence of damage. Modal shape changes seem to be more effective than natural frequency changes as damage indicators and they do not need always a mathematical model of the structure to locate damage. In [7–10], various applications are presented in which all of these methods usually seem to work properly with severe damage, but are too sensitive to the presence of noise. Moreover, several of the proposed vibration methods perform well in numerical simulations, but their performances vary when used in experimental test cases in such a way that no definite conclusion on their practical reliability can be drawn [11].

This paper presents the experimental verification of a new vibration method that was originally proposed in [12]. The vibration properties chosen to characterize the dynamics of the structure are derived from the theory of the proper orthogonal decomposition (POD) [13], which is emerging as a powerful experimental tool in structural dynamics. It provides a basis for the modal decomposition of an ensemble of functions, such as data obtained in the course of experiments. Its properties suggest that it is the preferred basis to use in various applications. The most striking of these is optimality: POD provides the most efficient way of capturing the dominant components of an infinite-dimensional process with only a finite number of (and, often, surprisingly few) modes [13]. The proper orthogonal modes (POMs) capture more energy per identified mode than any other orthogonal complete basis and can be easily identified for nonlinear systems. Moreover, the energy distribution between POMs, which is defined by the corresponding proper orthogonal value (POV), can be used to identify the most important modes [13–15]. The POD may be used to quantify spatial coherence in turbulence problems [13–15] and oscillating structures monitored by several sensors [16] and to determine the number of active state variables in a dynamic system. Extensive applications of POD to structural dynamics were carried out by Kappagantu and Feeny [17], Feeny [18], Azeez and Vakakis [19], Ma et al. [20], Georgiou and Schwartz [21], Kerschen and Golinval [22], and Kerschen et al. [23].

In the present work, the POD theory will be applied to assess the structural integrity of beams. POD is closely related to the singular-value decomposition [24] and the principal component analysis [25], methods that have already been used in damage detection. In the

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present work, the POD methods are applied with a dense grid of sensors that measure accelerations and can therefore provide the characteristic shape of the deformed structure during the steady-state dynamics. The changes in such a shape are used to detect and locate the damage, whereas in [24], the singular-value decomposition was used to detect damage in a structure with properties varying in time, and in [25] the principal component analysis was coupled with a statistical process control to enhance the discrimination between features of the undamaged and damaged structures.

The paper is organized as follows: Section II summarizes the main ideas of the POD from an engineering point of view. Section III describes the numerical model and the computational techniques used to simulate the experimental setup and the relevant numerical results. Section IV describes the experiment and the experimental acquisitions. Section V draws some conclusions.

II. Fundamental Ideas on the POD

The main objective of the present section is to give an elementary introduction to the fundamental concepts of the theory of the proper orthogonal decomposition applied to structural vibrations and structural damage detection. The application of POD to structures usually requires the experimental acquisition of displacements or accelerations at N locations of a vibrating system. In the examples presented in [12,26,27], displacement components are considered, whereas in the numerical examples in Sec. III and in the experimental application presented in Sec. IV, accelerations are used. Because the main emphasis of the present work is on the experimental application, we will refer to accelerations in the remainder of the present section. The recorded values of the accelerations at a given time t are labeled

$$a_1(t), a_2(t), \dots, a_N(t)$$

If the accelerations are sampled for M times simultaneously at the N locations, one can form acceleration-history arrays, such that

$$\boldsymbol{a}_i = (a_i(t_1), a_i(t_2), \dots, a_i(t_M))^T$$

for $i = 1, 2, \dots, N$. When performing the proper orthogonal decomposition, these displacement histories are shifted by subtracting the mean value $\bar{a}_i$. The vectors $\boldsymbol{A}_i$ are then formed as

$$\boldsymbol{A}_i = \boldsymbol{a}_i - \bar{a}_i \mathbf{1} \tag{1}$$

where $\mathbf{1}$ is a vector of dimension M for which the components are all equal to unity. The vectors $\boldsymbol{A}_i$, which therefore have zero mean, are then used to form an $M \times N$ matrix:

$$\boldsymbol{A} = [\boldsymbol{A}_1, \boldsymbol{A}_2, \dots, \boldsymbol{A}_N] \tag{2}$$

Each row in $\boldsymbol{A}$ is linked to the different accelerations of the recorded points of the structure at a particular instant in time; for that reason, it is often called a *snapshot*. Each column in $\boldsymbol{A}$, as previously noted, represents the zero-averaged time series of a single point of the structure. Matrix $\boldsymbol{A}$ is used to form the correlation matrix $\boldsymbol{R}$:

$$\boldsymbol{R} = \frac{1}{M} \boldsymbol{A}^T \boldsymbol{A} \tag{3}$$

where $\boldsymbol{R}$ is real and symmetric; therefore, its eigenvectors form an orthogonal basis. The eigenvectors of $\boldsymbol{R}$ are the POMs and the eigenvalues are the POVs of the system. POMs are characterized by particular optimality properties that make them suitable to represent the information contained in vectors $\boldsymbol{A}_i$ in an extremely efficient way. When applied to harmonically forced linear systems, the optimality features of POD appear to be particularly convenient; in fact, in [22], it was shown that

> *The forced harmonic response of a linear system is captured by a single POM*. Nevertheless, all of the linear natural modes are necessary to reconstruct the response. This property is independent of the mass distribution and underlines the optimality of the POMs.... The convergence of the POM to a natural mode is not guaranteed. The POM appears as a combination of all natural modes.

Therefore, the application of POD techniques to a harmonically forced linear system seems to take all of the information that is contained in an infinite number of linear natural modes and to *concentrate it in a unique POM*.

Preliminary tests (not documented in the present work) [12,26,27] seem to suggest that POD can also show very beneficial results when applied to nonlinear cases or nonharmonic forces, but in these cases, more than one POM will generally have to be considered. In the general nonlinear and/or nonharmonic case, not all of the POMs are significant, because some of them could carry a negligible amount of energy. There is the need for a criterion to identify the dominant mode shapes from the rest, a usual problem in modal approaches. Such a problem is easily solved with the POMs; this is achieved by using the POVs, which provide a measure of the strength of participation (or energy) of the corresponding mode to the signal. If λ_i indicates any POV and POVs are in decreasing order so that $\lambda_1 > \lambda_2 > \cdots > \lambda_N$, the number of dominant POMs is usually selected in such a way that the eigenvalues satisfy the following condition for the smallest integer p [21]:

$$\sum_{i=1}^{p} \lambda_i \Big/ \sum_{i=1}^{N} \lambda_i \geq 0.9999 \tag{4}$$

The main idea of the present work consists of choosing the changes in POMs as the damage indicator. The POMs are not only a property of the structure and its boundary conditions, as the linear natural modes, but they also depend on the dynamics of the system. If a harmonic force is applied to a point of a structure, the POMs of the structure change with the frequency of the excitation or with the point at which the force is applied. It is expected that they are similar to the linear natural modes when the frequency of the applied force is close to a natural frequency of the structure (see [12] for more details). The POMs of a beam in its undamaged and damaged states are computed, and the presence and location of damage are inferred from the changes induced in the POMs by the damage.

In the following sections, we will refer only to the first POM, because we assume that our systems are (almost) linear. Such an assumption is true in both the simulations and in the experiments, because the damage is a small saw cut that does not affect the linear elastic nature of the constitutive law. Therefore, the dynamic behavior of the system is fully captured by only one POM that contains (in its variations due to damage) the information about damage that will be used in the present work. Moreover, because the POM is defined along the axis of the beam, the position of its changes will also indicate the location of the damaged point of the structure.

The method can also be used when the damage is a fatigue crack that periodically closes and opens in the process of cyclic deformation of a structure (and, for this reason, it is often termed a closing, or breathing, crack), leading to the instantaneous change of structure stiffness. As a matter of fact, if the beam is excited around the first resonance, only one POM will be dominant, but more than one POM will have to be used in a general nonlinear case.

The same motions used to compute the dominant POMs in the undamaged structures are imposed on the damaged beams. In the next section, several figures will be plotted to show how the method works. The quantity chosen as the damage indicator is the difference ΔPOM, defined as

$$\Delta \text{POM}(i) = \text{POM}_u(i) - \text{POM}_d(i), \qquad i = 1, 2, \dots, N \tag{5}$$

where $\text{POM}_u(i)$ is the value of the dominant POM in the undamaged case at the ith sensor, and $\text{POM}_d(i)$ is the value of the dominant POM in the damaged case at the same node. The horizontal axis of the figures represents the length along the axis of the beam from which the accelerations at $N(=13)$ points are recorded.

The procedure presented in this paper, based on *proper orthogonal modes*, retains the main advantage of vibration techniques (i.e., monitors the changes in the structure on a global basis [2]. Moreover, it constitutes a significant step forward with respect to more traditional vibration techniques [11] based on the *linear natural modes* for three main reasons:

1) The new technique operates only on experimental data and no mathematical model of the structure is required.

2) The optimality of the proper orthogonal modes only requires the use of a few (only one, in the case of linear systems) POMs, whereas applications using the linear natural modes often require the evaluation of several modes. Damage, particularly in its initial phases, is a local phenomenon that does not significantly affect the low-frequency natural modes of a vibrating structure, whereas its effects are more noticeable on the higher modes [11]. For that reason, methods based on the use of linear natural modes face the problem of deciding how many natural modes have to be examined and which are the most relevant to detect the possible presence of damage. In contrast, the dominant POM is associated with the largest eigenvalue and is therefore uniquely determined. Moreover, such optimality seems to make the POMs more robust with respect to the unavoidable presence of noise [12,21].

3) The computation of POMs and POVs based on Eqs. (1–4) and on an eigenproblem solution is extremely simple, fast, and accurate. Moreover, no modal analysis of the structure is required.

III. Numerical Model of Damaged and Undamaged Beams

The modeled beam shown in Fig. 1 is a cantilever beam with a square cross section of $20 \times 20\ \text{mm}^2$ and a length of 520 mm, made of steel (AISI 1030), with the material properties as follows: $E = 203$ GPa, $\nu = 0.29$, and $\rho = 7850\ \text{kg/m}^3$. In the finite element method simulation, a proportional damping model was considered [28] and a modal damping ratio equal to 0.05 was chosen for the frequency range of interest. The black dots along the beam in Fig. 1 illustrate the locations at which the accelerations are computed to calculate the POMs and represent the positions of the accelerometers in the experimental test. MSC Patran was used to obtain the finite element model of the undamaged and damaged beams; numerical simulations of the dynamic behavior of the beams were performed using Nastran 2005.

An undamaged and two damaged beams were modeled and, for the latter case, the damage was created with a cut made at 190 mm from the clamped end, as shown in Fig. 2. The dimensions of the cuts, lying between the fifth and the sixth accelerometers, are also shown in Fig. 2.

It was decided to use a regular very fine mesh to catch the effect of the cut on the dynamic behavior of the beam. Consequently, HEXA8 solid elements were used in the modelization, considering 520 elements along the x axis, 40 along the z axis, and 4 along the y axis. To model the cuts, one row of elements was removed to simulate case A and four rows were removed to simulate case B. The model is linear because the cut interfaces are not in contact with each other during the application of the excitation force, due to the fact that the cuts made are relatively wide (1 mm), as shown in Fig. 2.

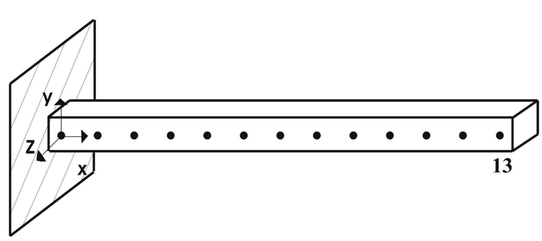


Fig. 1 Model of the beam; black dots represent the positions at which the accelerometers are located in the experimental setup.

The beam is excited with a force applied in correspondence to the 13th accelerometer in the z direction. The values of the accelerations in the same direction of the 13 accelerometers in Fig. 1 are used in the generation of the correlation matrix $\mathbf{R}$ used to compute the POMs.

The numerical simulations were performed considering 10,000 integration steps; the total simulated time of each case is 4.88 s, but of this interval, only the final 3 s were used to verify the method. The first natural frequency of cantilever beam was considered to decide the frequency of the excitation force to apply. In the undamaged case, it was equal to 60.800 Hz, and in the damage cases A and B, respectively, it was 60.794 and 60.715 Hz.

The ΔPOM obtained considering a sinusoidal excitation with frequency equal to 60 Hz is shown in Figs. 3a and 3b for the damage cases A and B, respectively. Evidently, for both damage cases, a sudden change of slope of the ΔPOM correctly indicates the position of the cuts. Furthermore, it is possible to note that the amplitude of the ΔPOM seems directly related to the extent of the damage.

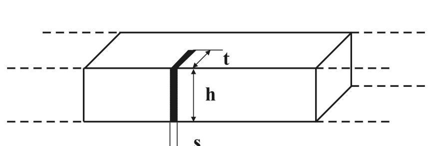
To investigate the sensitivity of the damage detection technique proposed, multiplicative noise (which is the more difficult type of noise to deal with) generated from a Gaussian distribution with a mean of 1 and a standard deviation of different levels was introduced into the acceleration data. A different noise sequence was applied to the acceleration time series of each recorded point. Although the proposed damage detection method is able to locate damage in the case of cut B for a multiplicative noise characterized by a standard deviation of up to 2% (see Figs. 4a and 4b), it was impossible to detect the position of cut A.

The main objective of the present work is to experimentally verify the proposed method. A more complete analysis of its performance based only on *numerical simulations* is presented in [12], in which the effects of varying the following quantities are investigated: 1) excitation frequency, 2) damage position, 3) damage severity, 4) position of the applied forcing, and 5) intensity of noise.

IV. Experimental Setup

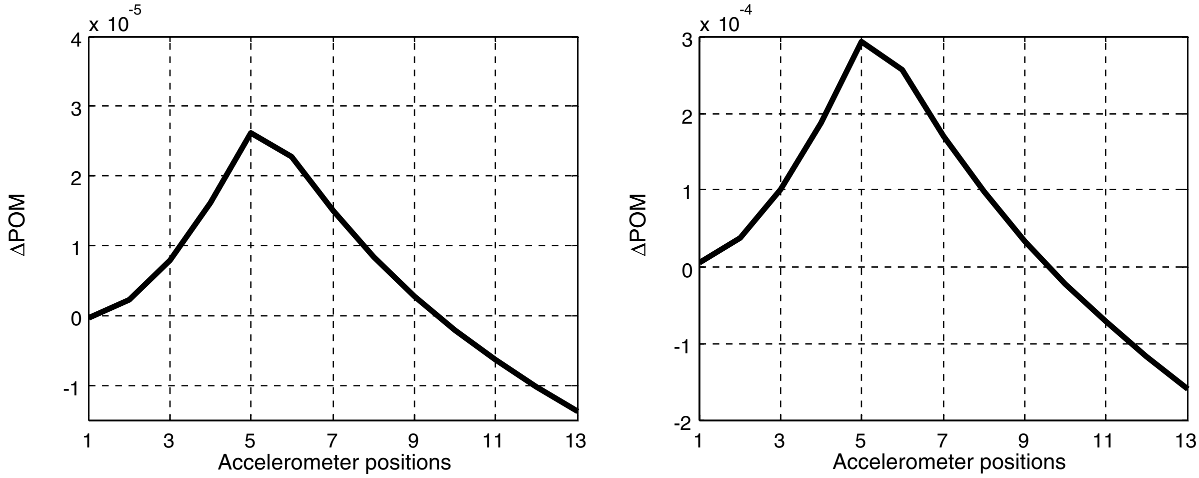
To evaluate the effectiveness of the method, experimental tests were performed on a steel cantilever beam with the same dimensions and material properties as the modeled structure. The first test was performed on the undamaged beam, for which the estimate of the first natural frequency was 60.78 Hz; the responses were acquired at 13 regularly spaced positions along the beam, in agreement with the numerical simulations, as is clearly visible in Figs. 5 and 6 (note that in Fig. 6, the first accelerometer is not visible). The shaker was placed in the two positions shown in Fig. 5, corresponding to accelerometers 1 and 13. For each shaker position, four different frequency values were used for the sinusoidal driving force: 40, 50, 60, and 90 Hz. Measurement time was set to 16 s and the sampling frequency was 1024 Hz.

After all of the acquisitions were performed, a small cut of 0.5-mm depth (damage case A in Fig. 2 and 6) was made with a saw between



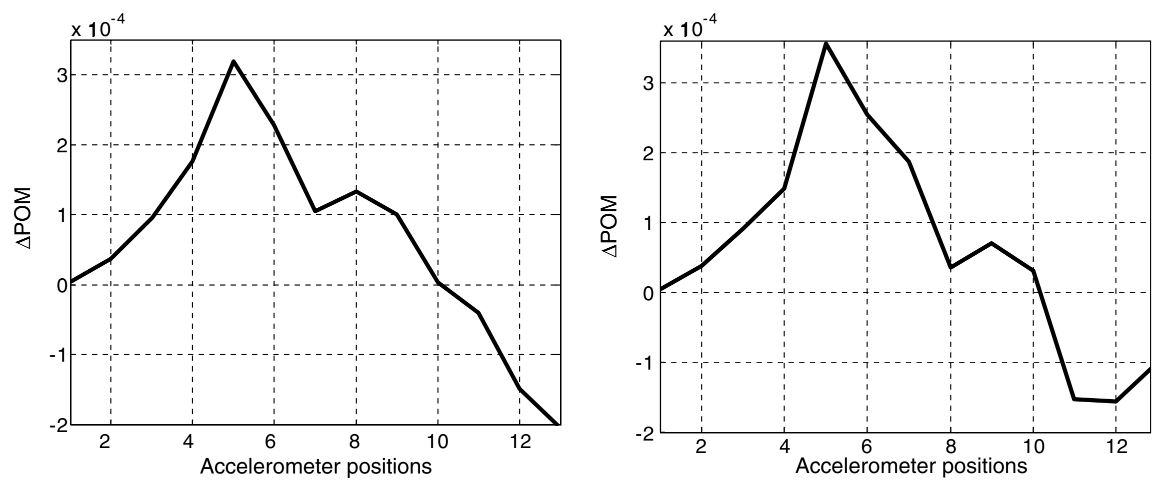
	s [*mm*]	**h** [*mm*]	**t** [*mm*]
CUT A	1	20	0.5
CUT B	1	20	2

Fig. 2 Dimensions of the cuts.



a) b)

Fig. 3 Numerical results, ΔPOM for the damaged beams without noise: a) cut A and b) cut B.



a) b)

Fig. 4 Numerical results, ΔPOM for the damaged beam with Gaussian noise, cut B: a) Gaussian noise with $\mathrm{std} = 1\%$, and b) Gaussian noise with $\mathrm{std} = 2\%$.

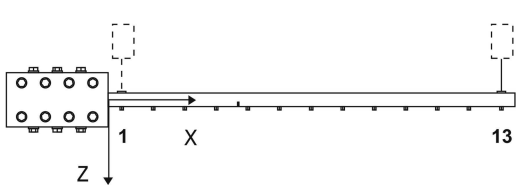


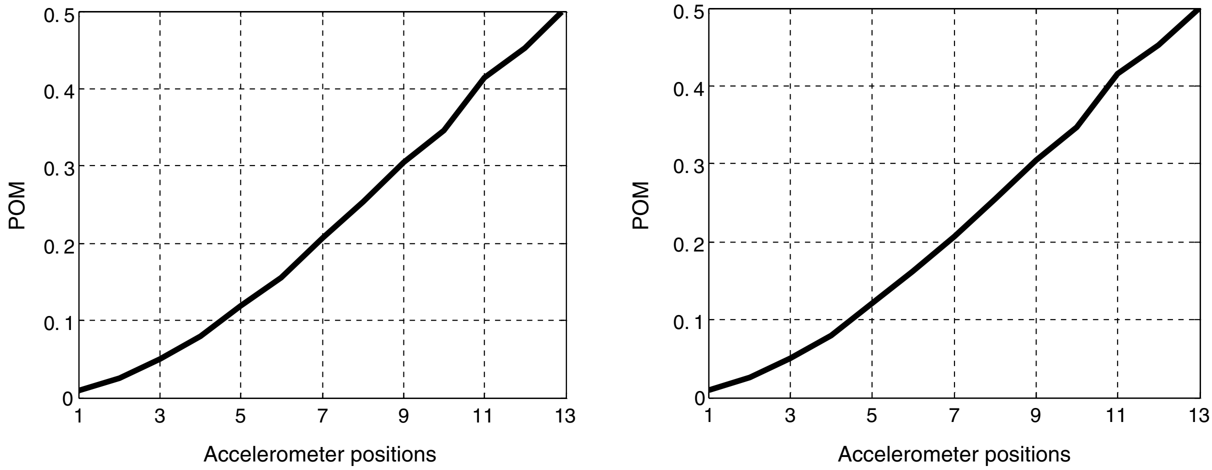
Fig. 5 Sketch of the experimental setup seen from above; two shaker positions and all accelerometers locations are shown.

the fifth and sixth nodes on the beam surface. The cut was so small that it did not affect the value of the first natural frequency in any significant way. As for the undamaged case, two shaker positions and four frequency values of the excitation were used to acquire data. Then the cut depth was increased to reproduce the damage case B shown in Figs. 2 and 6. In this case, the first natural frequency estimate was 60.69 Hz. It is apparent that such a small variation of the first natural frequency would be almost impossible to notice. This is the drawback of any damage detection method based on using natural frequencies.

Figures 7–10 present the results obtained experimentally, in which POMs were calculated after removing the transient part of measured signals (only the last 6 s were used in the evaluation of the POMs). The results shown in the present section were obtained with unfiltered data.

Figures 7a and 7b show the first POM for the undamaged and damaged beams, respectively (case A), using a frequency equal to 60 Hz, and the shaker is applied in correspondence to

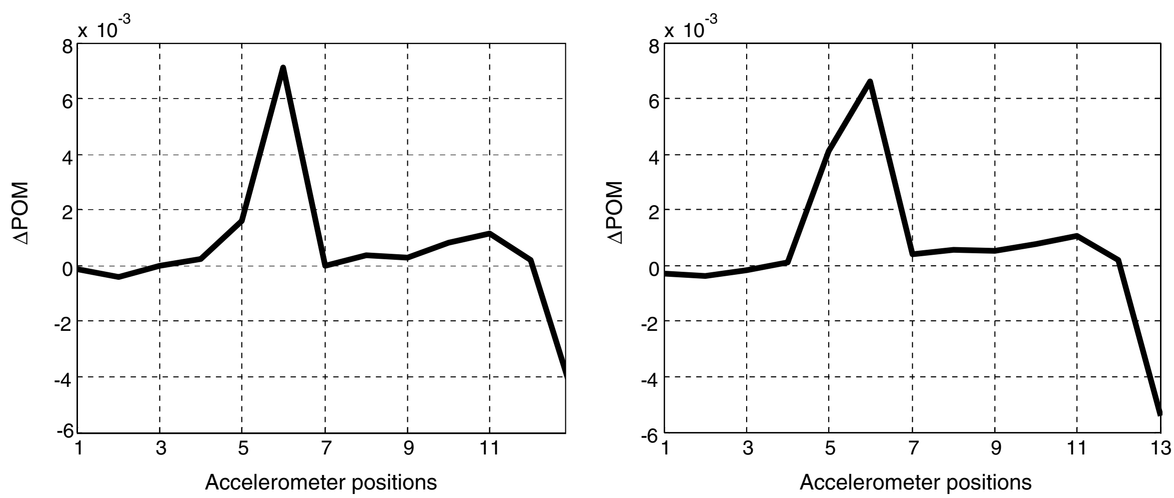
Fig. 6 Experimental setup showing the accelerometers and the shaker applied to the surface of the beam opposite to accelerometer 13; the larger saw cut (case B) is visible after accelerometer 5.



a) b)

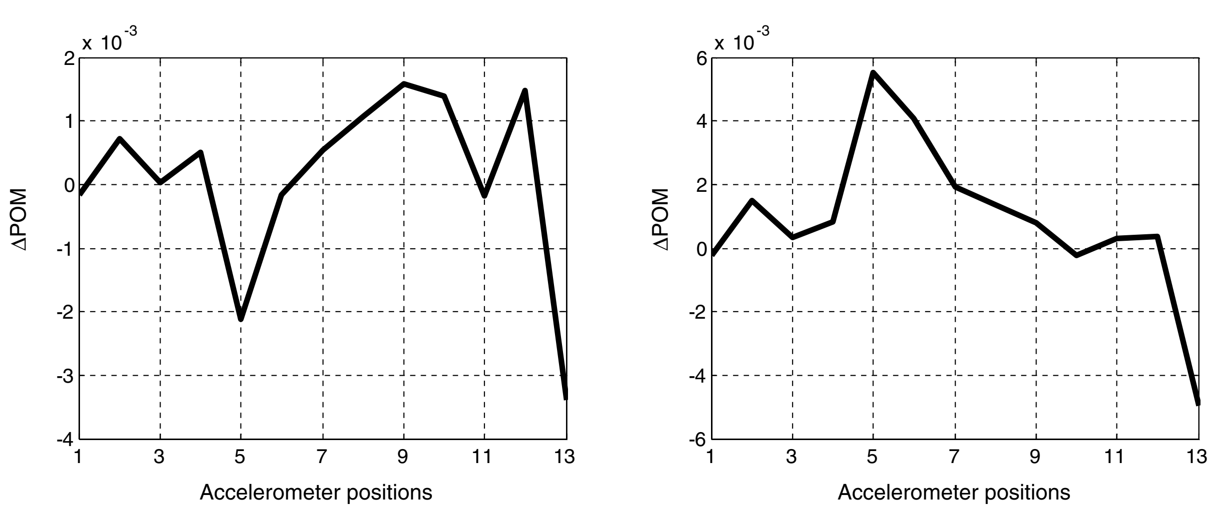
Fig. 7 Experimental results for excitation frequency of 60 Hz applied to position 13, cut A: a) undamaged and b) damaged.

accelerometer 13. It is not easy to infer the presence of damage from a simple visual comparison of the two graphs. However, the graph of the damage indicator ΔPOM gives a much clearer indication about the damage position. The experimental results in Figs. 8a and 8b show that the ΔPOM clearly detects the damage: a sharp peak appears in correspondence to the saw cut for the two considered cases. Even though the damage is correctly located, no clear correlation between the amplitude of the ΔPOM with the severity of



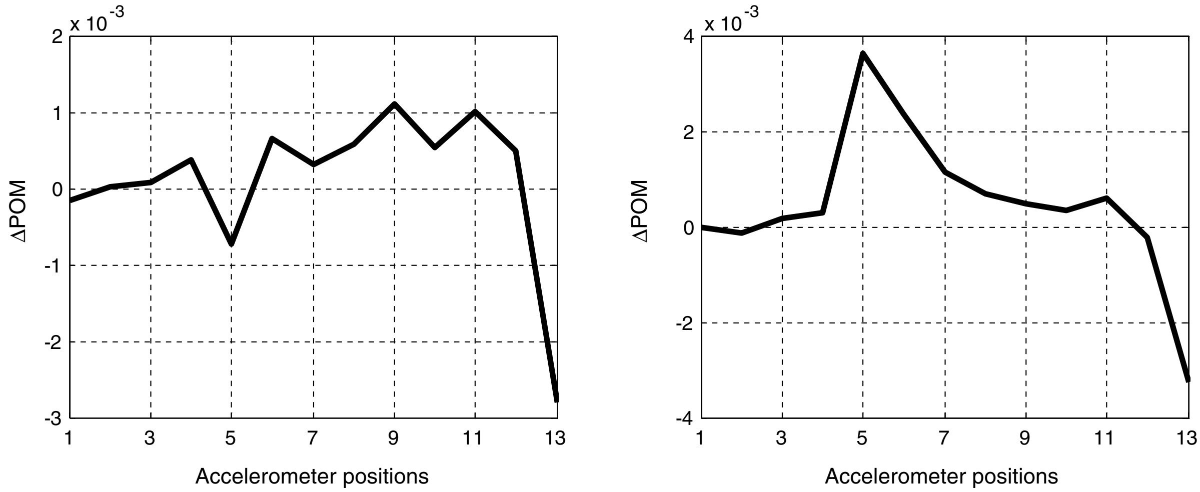
a) b)

Fig. 8 Experimental results for excitation frequency of 60 Hz applied to position 13: a) cut A and b) cut B.



a) b)

Fig. 9 Experimental results for excitation frequency of 40 Hz applied to position 13: (a) cut A and b) cut B.



a) b)

Fig. 10 Experimental results for excitation frequency of 50 Hz applied to position 13: a) cut A and b) cut B.

the damage, as in the numerical simulations, was found. Moreover, in this case, damage is indicated by a sharp peak rather than by a change in the slope of the ΔPOM diagram. The reason for such a discrepancy is still under investigation.

Using an excitation frequency different from 60 Hz, it was possible to correctly locate only the deeper cut (damage case B). Figures 9a, 9b, 10a, and 10b show the results obtained considering an excitation frequency equal to 40 and 50 Hz applied in correspondence to accelerometer 13 for both the damaged cases.

Finally, the effect of the position of the shaker was considered. Encouraging results are obtained when the excitation is applied in correspondence to accelerometer 13, as shown in the Fig. 8. It was noted that if the excitation is applied very close to the clamped end and the energy transmitted to the beam is very low as a consequence, the amplitude of the acceleration of the structure is not sufficiently high. For this reason, the measurements obtained with the shaker in correspondence to accelerometer 1 were neglected, because the ratio of signal to noise was too low to allow the method to be applied reliably.

All of the experimental graphs in Figs. 8–10 show a large negative value for ΔPOM at accelerometer 13. The reason for that is not clear and will be investigated in the future.

These preliminary experimental results suggest that the proposed method is promising because it is capable of locating a small damage in a beam with a compact cross section with no filtering of the recorded data. However, further research is required to have a better understanding of the following issues: 1) optimal range of frequencies to be applied, 2) optimal position at which the external excitation should be applied, 3) application to composite beams and/or plates, 4) use of filtering of the signal, and 5) use of impulsive excitation. Moreover, in the cases presented here, the first POM was sufficient for a precise localization of the damage. Systems affected by more serious nonlinearities could require the use of more than one POM for damage detection; in such a case, the concept of principal angles between subspaces formed by the POD modes could become useful [29,30].

Finally, a few limitations of the method have to be pointed out:

1) The method is not ideally suited for ambient excitation, at least in the case of strong random components in the ambient excitation. Because the POMs crucially depend on the external force, a change in it would cause changes in the POMs that would not necessarily be easily distinguishable from those generated by damage. However, the proposed method would find an ideal field of application in a controlled environment, such as in the case of quality control of an industrial production.

2) Damping should not constitute a major problem for the application of the method, at least as long as the energy provided to the structure by the external forcing generates a vibration of sufficient amplitude.

V. Conclusions

In this paper, the application of a structural damage detection method based on the use of *proper orthogonal modes* is presented. The technique has the advantage that it operates only on measured data, no mathematical model of the structure is needed, and it requires only a few (one in the case of linear systems) POMs, whereas applications using linear natural modes often require the evaluation of several modes. Furthermore, the computation of POMs based on Eqs. (1–4) and on an eigenproblem solution is extremely simple and fast, whereas the estimate of linear modes using experimental modal analysis involves the use of system identification algorithms that are less straightforward to apply and to interpret.

Two damage cases of a cantilever beam with a saw cut were studied both numerically and experimentally. In both cases, the damage was correctly located, even for the small cut. So the technique seems to be quite sensitive to small damage levels, even though the magnitude of the damage index ΔPOM was shown to be related to the cut extent only when numerical data were used.

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